

Transport in a Luttinger liquid with dissipation: Two impurities

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We consider theoretically the transport in a one-channel spinless Luttinger liquid with two strong impurities in the presence of dissipation. As a difference with respect to the dissipation free case, where the two impurities fully transmit electrons at resonance points, the dissipation prevents complete transmission in the present situation. A rich crossover diagram for the conductance as a function of applied voltage, temperature, dissipation strength, Luttinger liquid parameter K , and the deviation from the resonance condition is obtained. For weak dissipation and $1/2 < K < 1$, the conduction shows a nonmonotonic increase as a function of temperature or voltage. For strong dissipation the conduction increases monotonically but is exponentially small.

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I. INTRODUCTION

Once a quantum mechanical particle traveling in one-dimensional space hits a potential barrier formed by an impurity, it is reflected with a finite probability. This is not longer true in the case of two consecutive identical barriers forming a quantum dot: the waves reflected at the first and the second barrier may interfere destructively such that there is perfect transmission. The condition for the latter reads $\tan(ka) = -\hbar^2 k / (mU_0)$. Here m and k denotes the mass and the wavevector of the incoming wave and a is the spacing between the two barriers which are here assumed to be delta functions of the strength U_0 . For small and large U_0 this resonance corresponds to the conditions $ka = (n + 1/2)\pi$ and $ka = (n + 1)\pi$, respectively, where $n \geq 0$ is an integer. Thus, for large U_0 the resonance happens if the incident particle has the same energy as one of the bound states formed between the barriers, provided they are impenetrable.

So far we considered free electrons. In this paper we will examine the case of interacting electrons in one dimension where the interaction can be parameterized by a single parameter K with $K < 1$ ($K > 1$) for repulsive (attractive) interaction.^{1,2} Then, in the case of scattering at an isolated impurity, attractive (repulsive) interaction leads to perfect transmission (reflection) at low energies.³

If two consecutive strong impurities are present, the physics is influenced by the Coulomb blockade in the quantum dot. As was shown by Kane and Fisher^{4,5} and by Furusaki and Nagaosa,⁶ resonant transmission of spinless electrons is then possible provided $k_F a = (n + \frac{1}{2})\pi$, where k_F is the Fermi momentum and $n \geq 0$ is an integer. At resonance the ground state of the dot is doubly degenerate and has $n_{\pm} = q_0 \pm 1/2$ particles inside, where $q_0 = k_F a / \pi$ denotes the background charge between the two impurities. Note that this resonance condition is independent of the impurity strength U_0 in the interacting case.^{4,5} Tunneling through impurities is sequential at resonance. If one starts with the n_- state of the dot, adding a particle to the dot does not change the Coulomb energy. In second tunneling step the particle number in the dot goes back to n_- . In a similar way we may start with the n_+ state and then first decrease n_+ by one, which is followed by a

second electron tunneling into the dot to get back to the original state n_+ .

Thus under resonance conditions adding (or removing) an electron does not change the Coulomb energy inside the dot. Renormalization group analysis of the impurity strength then shows that perfect transmission is still present for weak impurities as long as $K > 1/4$.⁴⁻⁶ For strong impurities perfect transmission survives for $K > 1/2$. The conductance in both cases is given by e^2/h . Therefore, similar to the noninteracting case, the existence of a second impurity increases the tendency toward perfect transmission for not too strong repulsive interaction.

Adding an electron *off-resonance* is accompanied by an energy increase which has to be provided either by a thermal bath or by a finite external voltage. A thermal bath as well as a finite voltage drop across the dot also allows sequential tunneling off-resonance, which leads to power laws of the conductance as a function of temperature or voltage. If both are small enough, sequential tunneling is suppressed and tunneling occurs in one step via the formation of a virtual state in the dot (cotunneling).

In the present paper we want to study the influence of ohmic dissipation on the scenario presented so far. As discussed recently in Ref. 7, ohmic dissipation may result from the coupling of electrons in the Luttinger liquid to normal Fermi-liquid like electrons in nearby gates. Under the conditions considered in Ref. 7 coupling to the gate is relevant only for $K < K_{\eta} = 1/2$. However, other scenarios are conceivable, and in the following we will assume that K_{η} may take also larger values. We will therefore assume that dissipation is present. Clearly, for $K > K_{\eta}$ our results have to be replaced by those of the dissipation free case. Dissipation introduces a length scale $L_{\eta} \approx 1/(K\eta)$ where η denotes the dissipation strength. On scales larger than L_{η} the plasmon excitations of the electrons become diffusive and displacement fluctuations are strongly suppressed, restoring translational long-range order (Wigner crystal). If $Kv\eta \gg \omega$, the conductivity $\sigma_{\eta} = 2KL_{\eta}e^2/h$ is finite which is paralleled by diverging superfluid fluctuations. We have recently shown⁸ that dissipation has a dramatic influence on the tunneling of electrons through a single impurity, which is strongly suppressed. The voltage and temperature dependence of the conductance is

reduced from power laws in the dissipation free case to an exponential dependence for all $K < K_\eta$ at low energies. Thus there is no region of perfect transmission anymore with a single impurity. In the present paper we want to extend these considerations to the case of two impurities.

The paper is organized as follows. In Sec. II we present a model for spinless electrons in the presence of dissipation and with two impurities. Using the instanton method we study the electron tunneling through impurities driven by an external voltage or temperature. The conductance of the system is calculated in Sec. III for the cotunneling case, and in Sec. IV for the sequential tunneling. We conclude the paper in Sec. V. Some technical details are presented in Appendices.

II. TUNNELING PROBABILITY

A. Model

We consider a one-dimensional interacting system of spinless electron with two impurities coupled to a dissipative bath. The impurities are at positions $x = \pm a/2$. We will refer to the spacing between the impurities as the *quantum dot*. Using the standard bosonization methods,^{2,9} the Euclidean action for this system is

$$\begin{aligned} \frac{S}{\hbar} = & \int_{-L/2}^{L/2} dx \int_{-\hbar\beta/2}^{\hbar\beta/2} d\tau \left\{ \frac{1}{2\pi K} \left[\frac{1}{v} (\partial_\tau \phi)^2 + v (\partial_x \phi)^2 \right] \right. \\ & \left. + \rho \frac{U_0}{\hbar} [\delta(x - a/2) + \delta(x + a/2)] \right\} \\ & + \frac{\eta}{4} \int_{-L/2}^{L/2} dx \int_{-\hbar\beta/2}^{\hbar\beta/2} d\tau \int_{-\hbar\beta/2}^{\hbar\beta/2} d\tau' \frac{[\phi(x, \tau) - \phi(x, \tau')]^2}{\left[\hbar\beta \sin \frac{\pi(\tau - \tau')}{\hbar\beta} \right]^2}. \end{aligned} \quad (1)$$

Here the displacement field $\phi(x, \tau)$ is related to the electron density ρ by

$$\rho = \frac{k_F}{\pi} - \frac{1}{\pi} \partial_x \phi + \frac{k_F}{\pi} \cos(2\phi - 2k_F x) + \dots \quad (2)$$

The first term of action (1) is the well-known Tomonaga-Luttinger model. The parameter K measures the interactions between electrons, where $K < 1$ ($K > 1$) for repulsive (attractive) interaction between electrons. v is the velocity of the plasmon excitations. The second part in action (1) is the contributions from the two impurities, where we have assumed that the two impurities have the same strength U_0 .

The third piece describes Ohmic dissipation.¹⁰ It was shown in Ref. 7 that a dissipation term of the form

$$-\frac{\eta}{2} \int_{-L/2}^{L/2} dx \int_{-\hbar\beta/2}^{\hbar\beta/2} d\tau \int_{-\hbar\beta/2}^{\hbar\beta/2} d\tau' \frac{\cos[\phi(x, \tau) - \phi(x, \tau')]}{\left[\hbar\beta \sin \frac{\pi(\tau - \tau')}{\hbar\beta} \right]^2} \quad (3)$$

results from the coupling of the electrons in the wire to Fermi-fluid electrons in a nearby gate, where η is a coupling

constant. When the coupling is relevant, Eq. (3) can be expanded up to quadratic terms, which has been done in Eq. (1).

Integrating out the bulk phase field $\phi(x, \tau)$ except $\phi(x = -a/2, \tau)$ and $\phi(x = a/2, \tau)$, we obtain the effective action as

$$\begin{aligned} \frac{S}{\hbar} = & \frac{1}{2K\hbar\beta} \sum_{\omega_n} \left[\frac{1}{I_+(\omega_n)} |\phi_+(\omega_n)|^2 + \frac{1}{I_-(\omega_n)} |\phi_-(\omega_n)|^2 \right] \\ & + U \int d\tau \cos[\phi_+(\tau)] \cos[\phi_-(\tau) - k_F a], \end{aligned} \quad (4)$$

where

$$\phi_\pm(\tau) = \phi(a/2, \tau) \pm \phi(-a/2, \tau), \quad (5)$$

$$\phi_\pm(\omega_n) = \int d\tau e^{i\omega_n \tau} \phi_\pm(\tau), \quad \omega_n = \frac{2\pi n}{\hbar\beta}, \quad (6)$$

$$I_\pm(\omega_n) = \frac{\pi(1 \pm e^{-(a/v)\sqrt{\omega_n^2 + |\omega_n|v\eta K}})}{\sqrt{\omega_n^2 + |\omega_n|v\eta K}}, \quad (7)$$

and $U = 2k_F U_0 / (\pi\hbar)$ denotes the dimensionless pinning strength.

B. Classical ground state and excitations

We will first look at the *classical ground state*, where $\phi(x, \tau) \equiv \phi(x)$, which corresponds to the weak quantum fluctuation limit $K \ll 1$. In fact our further calculations are strictly justified only in this case although we will also apply our results for $K = \mathcal{O}(1)$. The field ϕ_- is related to the charge Q (in units of the elementary charge) accumulated between the two impurities by

$$Q = \int_{-a/2}^{a/2} dx \rho(x) = q_0 - \frac{\phi_-}{\pi}, \quad (8)$$

where $q_0 = k_F a / \pi$ denotes the background charge between the impurities. From Eq. (1), we now obtain

$$\frac{S_{\text{class.}}}{\hbar} = \frac{E_c}{2T} (Q - q_0)^2 + \frac{2k_F U_0}{\pi T} \cos \phi_+ \cos(\pi Q), \quad (9)$$

where the Coulomb energy of the quantum dot is

$$E_c = \frac{1}{\kappa a}, \quad \kappa = \frac{K}{\pi v \hbar}, \quad (10)$$

where κ denotes the compressibility. In the ground state, action (9) has to be minimal. For the rest of the paper we will assume strong impurities $k_F U_0 \gg E_c \gg T$. As a result Q has to be an integer. Then, minimizing the first term in Eq. (9), one obtains $Q = Q_0 = [q_0]_G$, where $[x]_G$ denotes the closest integer to x . Minimizing the second term gives $\phi_+ = (Q + 2m + 1)\pi$, where m is an integer. The corresponding ground-state values for $\phi(\pm a/2)$ are

$$\phi(-a/2) = \frac{1}{2} [(2m + 1)\pi - k_F a],$$

$$\phi(a/2) = \frac{1}{2}[(2m+1)\pi + k_F a] + \pi Q. \quad (11)$$

In the following it is convenient to express the background charge in the dot as

$$q_0 = n + \frac{1}{2} - \Delta, \quad (12)$$

where n is an integer and $-1/2 < \Delta < 1/2$. This gives $Q_0 = n$ if $\Delta > 0$ and $Q_0 = n+1$ if $\Delta < 0$. For $\Delta = 0$ the ground state is twofold degenerate, allowing $Q_0 = n$ and $Q_0 = n+1$, respectively.

As already observed by Kane and Fisher⁴ the action (9) remains invariant under the transformation $\phi_+ \rightarrow \phi_+ + 2\pi$. This transformation corresponds to the transfer of one electron from one to the other side of the dot. If q_0 is exactly half-integer, i.e., $\Delta = 0$, there is an additional invariance under the transformation $\phi_+ \rightarrow \phi_+ + \pi$ and $Q \rightarrow 2q_0 - Q$. This transformation corresponds to the transfer of one electron half-way across the double barrier structure. This allows the sequential tunneling through the quantum dot.

The energy cost for adding (removing) an electron to (from) the ground state is given by

$$E_{\pm} = \frac{E_c}{2}[1 \pm 2(Q_0 - q_0)] = E_c[\pm \Delta + \Theta_H(\mp \Delta)], \quad (13)$$

where $\Theta_H(x)$ denotes the Heaviside step function.

Let us now consider a sequential tunneling process in the case $1 \gg |\Delta| > 0$. For $\Delta > 0$ the ground state is given by $Q_0 = n$. To transfer an electron through the quantum dot it first goes from the left lead to the dot, which requires an energy $E_c \Delta$. In a second process it tunnels to the right lead which sets this energy again free. For $\Delta < 0$ the ground state of the dot is $Q_0 = n+1$ and the particle transfer begins with the tunneling of an electron from the dot to the right lead which costs $-\Delta E_c > 0$ followed by the tunneling of an electron from the left lead to the dot. Thus, for sequential tunneling with $\Delta \neq 0$ there is always one *hard* tunneling step which requires an energy $|\Delta|E_c$. This Coulomb blockade plays a role at non-zero temperatures T or voltages V_0 as long as $T, eV_0 < E_c|\Delta|$. Because of the symmetry between the cases $\Delta > 0$ and $\Delta < 0$, we can restrict ourselves in the following to the case $\Delta > 0$.

In addition to the charged excitations there are also *neutral* excitations in the quantum dot of spacing $\pi\hbar v/a = K/(\kappa a) = KE_c$. Their maximum energy is $\hbar\omega_c \approx \hbar v\Lambda$, where Λ is of the order of the Fermi momentum k_F . If the temperature T is larger than KE_c the quantization of the neutral excitations in the quantum dot becomes irrelevant and the tunneling through the two impurities will become independent, i.e., the tunneling is incoherent sequential. Alternatively, one can say that the coherence length $L_T = \hbar v/T$ of the displacements of the electrons in the quantum dot is smaller than a . In the opposite limit the tunneling is coherent. In the following we will concentrate on this case.

In our model the neutral excitations are damped. Plasmons become diffusive on scales larger than $L_\eta = 1/(K\eta)$, having a characteristic life time L_η/v . Phase coherence in the

quantum dot is lost if $L_\eta < a$, i.e., if $KE_c < \Gamma = \hbar v K \eta$ where Γ denotes the imaginary part of the plasmon energy.

C. Instanton action

The tunneling rate \mathcal{R} through the impurity and hence the current $I = e\mathcal{R}$ can be calculated from the imaginary part of the free energy

$$\mathcal{R} = \frac{2}{\hbar\beta} \text{Im} \ln Z, \quad (14)$$

where the partition function is

$$Z = Z_0 + iZ_1 = \int \mathcal{D}\phi_+(\tau) \mathcal{D}\phi_-(\tau) e^{-S[\phi_+, \phi_-]/\hbar}. \quad (15)$$

The real part of Z includes the (stable) fluctuations around the classical ground state. Since the imaginary part Z_1 of the partition function is small compared with the real part Z_0 , the tunneling rate can be written as

$$\mathcal{R} \approx \frac{2}{\hbar\beta Z_0} \text{Im} \int \mathcal{D}\phi_+(\tau) \mathcal{D}\phi_-(\tau) e^{-S[\phi_+, \phi_-]/\hbar}, \quad (16)$$

where the functional integral is with respect to the functions $\phi_{\pm}(\tau)$ defined on the interval $[-\hbar\beta/2, \hbar\beta/2]$ and satisfying $\phi_{\pm}(-\hbar\beta/2) = \phi_{\pm}(\hbar\beta/2)$. To calculate Z_1 we follow a method developed by Callan and Coleman.¹¹ There exist the saddle-point functions $\tilde{\phi}_{\pm}(\tau)$ which obey the equations

$$\left. \frac{\delta S[\phi_+, \phi_-]}{\delta \phi_{\pm}} \right|_{\phi = \tilde{\phi}} = 0. \quad (17)$$

With $\phi_{\pm}(\tau) = \tilde{\phi}_{\pm}(\tau) + \delta\phi_{\pm}(\tau)$ the action close to the saddle point trajectory can be written in the form

$$\begin{aligned} & S[\phi_+, \phi_-] - S[\tilde{\phi}_+, \tilde{\phi}_-] \\ & \approx \sum_{i,j=\pm} \frac{1}{2} \int d\tau d\tau' \left. \frac{\delta^2 S}{\delta \phi_i(\tau) \delta \phi_j(\tau')} \right|_{\phi = \tilde{\phi}} \delta\phi_i(\tau) \delta\phi_j(\tau') \\ & = \sum_{\substack{i,j=\pm \\ n,m}} V_{i,n;j,m} a_{n,i} a_{m,j} = \sum_n \lambda_n c_n^2, \end{aligned} \quad (18)$$

where in the last step we have expanded $\delta\phi_{\pm}(\tau)$ into a complete set of orthogonal functions $\psi_n(\tau)$, $\delta\phi_{\pm}(\tau) = \sum_n a_{n,\pm} \psi_n(\tau)$ and then diagonalized the resulting quadratic form in the $a_{n,\pm}$. One of the eigenvalues, $\lambda_{n=0}$, has to be negative to give rise to the imaginary part. Thus

$$iZ_1 = e^{-S[\tilde{\phi}_+, \tilde{\phi}_-]/\hbar} \mathcal{N} \int \left(\prod_n dc_n \right) \exp\left(-\sum_n \lambda_n c_n^2/\hbar\right). \quad (19)$$

One of the eigenvalue has to be zero corresponding to a shift of the instanton in the τ direction¹² delivering a factor $\hbar\beta$. Z_0 can be calculated in the same way with only positive eigenvalues at the stable saddle point.

Performing this program is very difficult in the present case. Instead, we will look for an approximate solution. In this case the saddle point function $\tilde{\phi}(\pm a/2, \tau)$ will assume

their ground state values (11) everywhere apart from the regions where $\tilde{\phi}(\pm a/2, \tau)$ is increased by π . This advancement of π is triggered by the applied external voltage. The connection between these pieces are narrow kinks and antikinks of width $\delta \sim 1/U \ll aK/v$. Further, we have to assume that the width of the kinks is always smaller than the typical kink-antikink spacing, which implies that the impurity is sufficiently strong (depending on the applied voltage). Thus the saddle-point configuration $\tilde{\phi}_{\pm}$ is determined in our approximate scheme by the positions of the kinks and antikinks. A kink-antikink pair will be called an *instanton* in the following.

It is sufficient to consider the case when there is only one instanton at each impurity with kink-antikink spacing equal to τ_1 and τ_2 on the left and the right impurity, respectively. To minimize the action we will assume that the centers of the instantons have the same value of τ . With our parametrization the saddle point $\tilde{\phi}_{\pm}$ is now found from the condition for the instanton action $\partial S_{\text{inst}}(\tau_1, \tau_2) / \partial \tau_{1,2} |_{\tau_i = \tau_{i,s}} = 0$. This gives

$$I = e\mathcal{R} \approx 2e^{-S_{\text{inst}}(\tau_{1,s}, \tau_{2,s})/\hbar} \text{Im} \int' d\vartheta_1 d\vartheta_2 \times \exp\left(-\frac{1}{2\hbar} \sum_{i,j=1,2} \frac{\partial^2 S(\tau_1, \tau_2)}{\partial \tau_i \partial \tau_j} \Big|_{\tau_i = \tau_{i,s}} \vartheta_i \vartheta_j\right), \quad (20)$$

where ϑ_i denotes deviations from the saddle point. Here we used $Z_0 \approx 1$. The prime at the integral excludes the integration over the center of mass of the instanton. At zero temperature this integral can be evaluated using the method introduced in Ref. 11. It gives a power-law correction. Unfortunately, at finite temperature we are not able to evaluate it, although we expect it to be also a power law. In the dissipation free case this correction is essential for a correct power-law exponent. In the dissipative case it is less important since the exponential part in Eq. (20) gives a dominating exponential dependence.

Below we will consider only two cases: either instantons of equal size appear at both impurities, corresponding to $\tilde{\phi}_{-} = 0$, i.e., $\tau_{1,s} = \tau_{2,s} \equiv \tau_s$. This case will be called *cotunneling*. Or there is only one instanton either on the left or the right impurity, corresponding to $\tilde{\phi}_{+} = \pm \tilde{\phi}_{-}$, i.e., $\tau_{1(2),s} \equiv \tau_s > 0$ and $\tau_{2(1),s} = 0$. This case will be called *sequential tunneling*. Using the results obtained previously for the single impurity⁸ case, the instanton action at $T=0$ can be written down immediately. It takes the form

$$\frac{S_{\text{inst}}}{\hbar} = \frac{2S_{\text{kink}}}{\hbar} [\Theta_{H,\delta}(\tau_1) + \Theta_{H,\delta}(\tau_2)] + \frac{2}{K} [f(\tau_1) + f(\tau_2)] - \frac{eV_0}{2\hbar} (\tau_1 + \tau_2) + |\tau_1 - \tau_2| E_{\text{sign}(\tau_1 - \tau_2)} \frac{1}{\hbar}, \quad (21)$$

where $\Theta_{H,\delta}(x)$ is a step function of width δ . The different terms have the following meaning: S_{kink} denotes the action of a kink, while $t \sim e^{-S_{\text{kink}}/\hbar}$ is the tunneling transparency of a single impurity. The next term in Eq. (21) includes the kink-antikink interaction with

$$f(\tau) = \int_0^{\omega_c} d\omega \frac{\pi}{I_+(\omega)\omega^2} [1 - \cos(\omega\tau)]. \quad (22)$$

The following voltage term describes the decrease in the energy by transferring an electron from the left to the quantum dot and from there to the right. Note that the voltage applied to at the ends of the system is, in general, different from the voltage at the impurities. However, if the wire is not too long and the impurities are strong, both voltages are approximately the same. Finally, the last term is the contribution from the Coulomb blockade. Going over to dimensionless time variable $\tau KeV_0/\hbar = y$ the action can be rewritten in the form

$$\frac{S(y_1, y_2)}{\hbar} = \frac{2S_{\text{kink}}}{\hbar} [\Theta_{H,\delta}(y_1) + \Theta_{H,\delta}(y_2)] + \frac{2}{K} [F(y_1) + F(y_2)] - \frac{1}{2K} (y_1 + y_2) + \frac{1}{K} |y_1 - y_2| \Delta X, \quad (23)$$

where

$$F(y) = \int_0^{yZ} \frac{d\Omega}{\Omega} \frac{\sqrt{1 + \frac{yY}{|\Omega|}} (1 - \cos \Omega)}{1 + \exp\left(-\frac{\pi\Omega}{yX} \sqrt{1 + \frac{yY}{|\Omega|}}\right)}, \quad (24)$$

with

$$X = \frac{E_c}{eV_0}, \quad Y = \frac{\Gamma}{KeV_0}, \quad Z = \frac{\hbar\omega_c}{KeV_0}. \quad (25)$$

Here we have used the ratios X , Y , and Z of the relevant energy scales of the problem. To calculate the integral we will assume that always $1, X, Y \ll Z$, i.e., $KeV_0, \Gamma, KE_c \ll \hbar\omega_c$. The integral can be approximated by the replacement $1 - \cos \Omega \approx \Theta_H(\Omega - 1)$. The calculation is done in Appendix B and gives the final result (B6).

D. Finite temperatures

So far we considered the case of zero temperature. At low but finite temperature the action and its saddle points are essentially unchanged, as long as the saddle point for τ is smaller than \hbar/T . For larger T the tunneling rate is determined from the maximum of the action taken at $\tau = \hbar/T$.¹³ Again, for sequential tunneling one of the saddle points of $\tau_{i,s}$ vanishes. For the further discussion it is convenient, instead of Eq. (25) to present the following dimensionless parameters:

$$X_T \equiv \frac{KE_c}{T}, \quad Y_T \equiv \frac{\Gamma}{T}, \quad Z_T \equiv \frac{\hbar\omega_c}{T}. \quad (26)$$

Accordingly, the dimensionless imaginary time is redefined as $z = \tau T/\hbar$. This gives, instead of Eq. (23), for the instanton action

$$\begin{aligned} \frac{S(z_1, z_2)}{\hbar} &= \frac{2S_{\text{kink}}}{\hbar} [\Theta_{H,\delta}(z_1) + \Theta_{H,\delta}(z_2)] - \frac{eV_0}{2T}(z_1 + z_2) \\ &+ \frac{2}{K} [F_T(z_1) + F_T(z_2)] + |z_1 - z_2| \Delta \frac{X_T}{K}, \end{aligned} \quad (27)$$

where $F_T(z)$ is given by Eq. (D1). Note that here either $z_1 = z_2 = 1$ for cotunneling or $z_1 = 1$ and $z_2 = 0$ (or vice versa) for sequential tunneling. The expressions for the currents at finite temperatures are derived in Appendix D.

E. Cross-over between sequential and cotunneling

As mentioned already, to find the tunneling rate and hence the current I , we have to calculate the saddle points $y_{1,s}, y_{2,s}$ of Eq. (23). At $T=0$ the result depends on X, Y, Z as well as on Δ and K . All terms in Eq. (23) are symmetric in y_1, y_2 apart from the last one which determines the difference between y_1 and y_2 . These saddle points are calculated in Appendix C in Eqs. (C1) and (C2), respectively. To find the cross-over line between sequential tunneling and cotunneling we have to equate the saddle-point action of the two cases:

$$2F(y_s) - \left(\frac{1}{2} - \Delta X\right) y_s = \frac{2KS_{\text{kink}}}{\hbar} + 4F(y_c) - y_c. \quad (28)$$

As it is shown in Appendix C the crossover between sequential tunneling and cotunneling happens at

$$X_c \equiv \frac{E_c}{eV_0} = \frac{1}{2\Delta} \begin{cases} \frac{1}{1+2Y_1}, & \frac{\Gamma}{KeV_0} = Y \leq 1 \\ 1 - \frac{1}{2 + KS_{\text{kink}}/(\hbar\pi Y)}, & \frac{\Gamma}{KeV_0} = Y \gg 1, \end{cases} \quad (29)$$

where $Y_1 \equiv \left(\frac{t^2 K}{2\Delta Z^2}\right)^{1/(1-K)}$. The crossover line is depicted in Fig. 1.

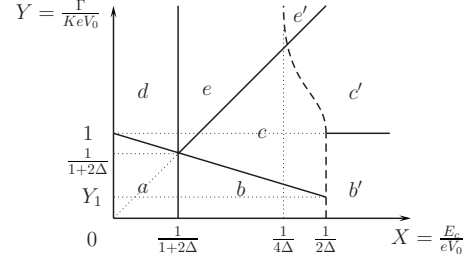


FIG. 1. Crossover diagram at zero temperature. Regions (a)–(e) correspond to sequential tunneling, while regions (b'), (c'), and (e') to cotunneling. The dashed crossover line is given by Eq. (29)

Next we calculate the crossover between cotunneling and sequential tunneling for finite temperatures. In this case the crossover condition corresponding to Eq. (28) is given by

$$\frac{2}{K} F_T(1) + \Delta \frac{X_T}{K} = \frac{2S_{\text{kink}}}{\hbar} + \frac{4}{K} F_T(1). \quad (30)$$

To solve this equation, we start with the regime $\Gamma, E_c K \ll T$, in which $F_T(1)$ is given by the expression case (i) in formula (D1). In this regime Eq. (30) leads to

$$\Delta X_T = \frac{2KS_{\text{kink}}}{\hbar} + 2 \ln Y_T \gg 2. \quad (31)$$

This violates the starting condition $\Gamma, E_c K \ll T$. Thus the crossover between the sequential tunneling and cotunneling is not possible in this regime. Similarly, it can be shown that the crossover cannot happen in the regime $X_T^2/Y_T \ll 1 \ll Y_T$. In the remain three regimes we find self-consistent solutions for the crossover. These results are summarized by the following expression

$$\Delta X_T \approx \begin{cases} 2KS_{\text{kink}}/\hbar, & Y_T \ll 1 \ll X_T \\ 2KS_{\text{kink}}/\hbar + \sqrt{2\pi}(\sqrt{Y_T} - 1), & 1 \ll Y_T \ll X_T \\ 2KS_{\text{kink}}/\hbar + [\sqrt{2\pi}Y_T + \sqrt{2\pi}(Y_T/X_T - 2)], & 1 \ll X_T^2/Y_T \ll Y_T. \end{cases} \quad (32)$$

The various regimes and crossovers between them for finite temperature are illustrated in Fig. 2.

III. COTUNNELING

A. Zero temperature

In this section we will consider cotunneling. In this case the instanton covers both impurities, $y_1 = y_2 = y_c$, and the electron will tunnel in one step through them.

In the regime of very weak dissipation, $\Gamma \ll KeV_0$, i.e., regime (b'), we get

$$I \sim t^4 \omega_\delta \left(\frac{KE_c}{\hbar\omega_c}\right)^{2/K} \left(\frac{eV_0}{\hbar\omega_c}\right)^{(2/K)-1}, \quad \Gamma \ll KeV_0 \ll KE_c \quad (33)$$

where $\omega_\delta \equiv 1/(\omega_c \delta^2)$ and $\delta \sim 1/U$ is a short time cutoff. This result is similar to the dissipation free case considered for a single impurity by Kane and Fisher.^{3,4} This is intuitively expected since in the cotunneling process the island can be effectively viewed as a “big” impurity with renormalized strength. The factor t^4 corresponding to the tunneling through two impurities. Such a t^4 prefactor has been found previously in a study of Coulomb blockade in a system with

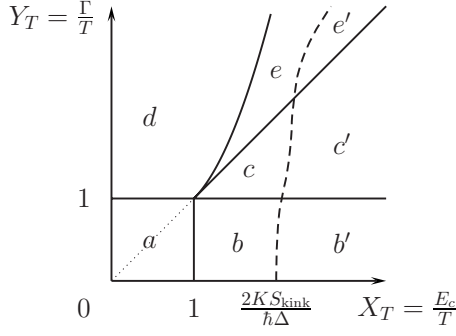


FIG. 2. Cross-over diagram at nonzero temperature. Regions (a)–(e) correspond to sequential tunneling, while regions (b'), (c'), and (e') to cotunneling. The dashed crossover line is given by Eq. (32).

long range interaction.¹⁴ For $K > 1$ the conductance $G = I/V_0$ diverges according to Eq. (33) which signals the approach to the perfect conductance $G = e^2/h$.

In the opposite regimes (c') and (e') of strong dissipation, we find

$$I \sim t^4 \omega_\delta e^{-2\pi\Gamma/K^2 eV_0} \left(\frac{\Gamma}{eV_0}\right)^{3/2} \left(\frac{\Gamma}{\hbar\omega_c}\right)^{(4/K)-1} (e^{\sqrt{2\pi}\Gamma/E_c K} - 1)^{-2/K},$$

$$KeV_0 \ll \Gamma, KE_c. \quad (34)$$

Again, this result has the same voltage dependence as those of the single impurity cases.⁸ As it follows from Eq. (34), dissipation strongly reduces the tunneling probability through the impurities. The last factor is an interpolation formula between the cases $\Gamma \ll KE_c$ and $\Gamma \gg KE_c$, respectively. Since this factor appears as well in the formulas below, $t^2(e^{\sqrt{2\pi}\Gamma/E_c K} - 1)^{-1/K} \approx t^2 e^{-\sqrt{2\pi}\Gamma/E_c K}$ can be considered as the effective transmission coefficient of the impurity in the case of strong dissipation.

B. Finite temperatures

For nonzero temperature, besides KE_c , Γ , $\hbar\omega_c$, and KeV_0 , the temperature T appears as a energy scale. This allows in general a large number of different regimes. In the following we will therefore restrict ourselves to the case where KeV_0 is smaller than all other energies.

In the limit of weak dissipation [regime (b')] we get for the conductance

$$G \sim t^4 \omega_\delta \left(\frac{T}{\hbar\omega_c}\right)^{(2/K)-2}, \quad \Gamma \ll T \ll KE_c \quad (35)$$

which again agrees, apart from the factor t^4 , with the result of Kane and Fisher³ in the dissipation free case. In the opposite regime (e') of strong dissipation we get instead

$$G \sim t^4 \omega_\delta e^{-\sqrt{8\pi}\Gamma/K^2 T} (e^{\sqrt{2\pi}\Gamma/E_c K} - 1)^{-2/K}, \quad T \ll KE_c, \Gamma. \quad (36)$$

It should be noted that the present approach does not allow the precise determination of the numerical prefactors in the

exponential terms. The leading temperature dependence is the same as for the single impurity case, see Ref. 8.

IV. SEQUENTIAL TUNNELING

A. Zero temperatures

Let us next consider the resonant case $\Delta=0$ where the cross-over line between sequential and cotunneling moves to infinity. Then, according to Eq. (13), starting with the ground state $Q=m$, it does not cost energy to add a particle to the quantum dot. To remove it from the quantum dot in the state $Q=m+1$ does not cost energy as well. The tunneling rate for each process is the same and follows from the saddle point of Eq. (23) with $y_2=0$ and $y_1=y_\delta$. Below we will present more general results for the case which includes a weak deviation from the perfect resonance.

If $0 < \Delta \ll 1$, bringing an electron to the quantum dot costs an energy ΔE_c whereas in the second step, in which the electron leaves the dot, this energy is again released. Thus tunneling through the dot is dominated by the first step. Similarly, if $\Delta < 0$, it costs first an energy $|\Delta|E_c$ to bring an electron out of the quantum dot whereas in the second step a second electron tunnels from the left into the dot, which is accompanied by an energy gain $-|\Delta|E_c$. Thus, again the second process is faster than the first one, the latter dominates the tunneling probability. Both cases can be combined by replacing Δ by its absolute value.

Plugging the expressions for the saddle points (C1) into the action we get for the current in regime (a) of large voltage

$$I = GV_0 \sim t^2 \omega_\delta \left(\frac{eV_0}{\hbar\omega_c}\right)^{(2/K)-1}, \quad \Gamma, KE_c \ll KeV_0. \quad (37)$$

This corresponds to nondissipative incoherent sequential tunneling and has the same voltage dependence as the single impurity tunneling in the absence of dissipation.³ The critical K value for the conductance G is $K=1$. For $K > 1$ the conductance increases for decreasing voltage signaling the perfect conductance in the zero voltage limit. It should however be taken into account that this limit cannot be performed because of the restriction $KeV_0 \gg \Gamma$.

In the opposite limit of very low voltage [regimes (c) and (e)] we get instead

$$I \sim t^2 \omega_\delta e^{-\pi\Gamma/K^2(eV_0-2|\Delta|E_c)} \left(\frac{\hbar\omega_c}{eV_0-2|\Delta|E_c}\right)^{3/2} \times (e^{\sqrt{2\pi}\Gamma/E_c K} - 1)^{-1/K}, \quad K(eV_0-2|\Delta|E_c) \ll KE_c, \Gamma. \quad (38)$$

In these cases for $|\Delta|=0$ the system shows dissipative resonant tunneling. In comparison with the corresponding result for the cotunneling (34), the expressions in Eq. (38) are larger by an exponent 1/2 in the leading voltage dependence (provided $\Delta \rightarrow 0$). Clearly, for all values of K dissipation is dominant and reduces the current strongly.

Finally, there are the intermediate cases (b)

$$I \sim t^2 \omega_\delta \left(\frac{KE_c}{\hbar\omega_c}\right)^{1/K} \left(\frac{eV_0-2|\Delta|E_c}{\hbar\omega_c}\right)^{(1/K)-1},$$

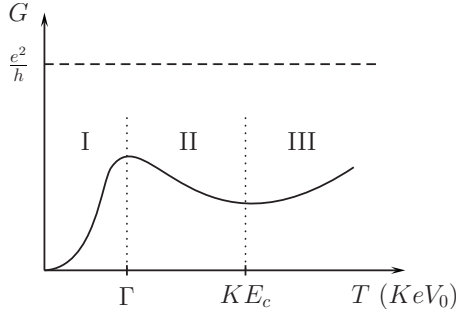


FIG. 3. Conductance as a function of temperature (voltage) for the case of weak dissipation $\Gamma \ll KE_c$.

$$\frac{\Gamma}{K} \ll eV_0 - 2|\Delta|E_c \ll E_c \quad (39)$$

and (d)

$$I \sim t^2 \omega_\delta \left(\frac{\Gamma}{\hbar \omega_c} \right)^{(2/K)-1} \left(\frac{\Gamma}{eV_0} \right)^{3/2} e^{-4\pi\Gamma/K^2 eV_0}, \quad E_c \ll eV_0 \ll \frac{\Gamma}{K}. \quad (40)$$

In case (b), Eq. (39), under resonant conditions, $|\Delta|=0$, the conductance $G=I/V_0$ diverges for $V_0 \rightarrow 0$ and $1/2 < K$, signaling the perfect conductance. For small but finite Γ , however, the conductance is limited by $G \sim t^2 \omega_\delta (\Gamma/\hbar \omega_c)^{1/K-2}$. Case (d) corresponds to dissipative incoherent sequential tunneling. In comparison with the dissipative single impurity result,⁸ the expressions in Eq. (40) are smaller by an exponent 2 in the leading voltage dependence.

Since in regimes (a) and (d) the tunneling through the two impurities is independent, the total conductance can be calculated by the formula for two identical conductances connected in series, $G=G_s(V_0/2)/2$, where $G_s(V_0/2)$ denotes the conductance for single impurity with voltage drop $V_0/2$. This also leads to results (37) and (40).

One can notice that in some formulas [e.g., Eqs. (34) and (40)] no factor Δ appears, while in Eqs. (38) and (39) it appears. The reason is that the latter two equations correspond to regimes (b), (c), and (e), see Fig. 1. In these regimes, $2\Delta E_c$ can be arbitrary close to eV_0 when one is close to the crossover between sequential tunneling and cotunneling. Formula (40) corresponds to regime (d), which is always far away from the crossover. This implies that ΔE_c is always much smaller than eV_0 , and hence neglected in Eq. (40). Equation (34) is for cotunneling regime where the Coulomb blockade term [i.e., the term that involves Δ in Eq. (23)] disappears.

B. Finite temperature

At finite temperature we obtain various tunneling regimes which have one to one correspondence to those at zero temperature. Moreover, the discussions for these regimes apply in both cases of zero and nonzero temperatures. Therefore, to avoid unnecessary repetition we only give results for finite temperature. We obtain for the conductance in regime (a)

$$G \sim t^2 \left(\frac{T}{\hbar \omega_c} \right)^{(2/K)-2}, \quad \Gamma, E_c K \ll T, \quad (41)$$

which has the same temperature dependence as the single impurity in the absence of dissipation.³

At low temperature we obtain the result of dissipative resonant tunneling of regimes (c) and (e)

$$G \sim t^2 e^{-E_c|\Delta|/T} A_3(T) e^{-\sqrt{2\pi\Gamma/K^2 T}} (e^{\sqrt{2\pi\Gamma/E_c K}} - 1)^{-1/K},$$

$$T \ll KE_c, \frac{(KE_c)^2}{\Gamma}, \Gamma. \quad (42)$$

$A_3(T)$ is some power-law temperature-dependent function, which is subdominant to the exponential temperature-dependent part in Eq. (42).

At intermediate temperature we find in regime (b)

$$G \sim t^2 \left(\frac{KE_c}{\hbar \omega_c} \right)^{1/K} e^{-E_c|\Delta|/T} \left(\frac{T}{\hbar \omega_c} \right)^{(1/K)-2}, \quad \Gamma \ll T \ll KE_c. \quad (43)$$

Thus the conductance in regimes (c), (e), and (b) is exponentially suppressed away from resonance. At resonance (i.e., $|\Delta|=0$) the conductance in regime (b) increases with decreasing temperature if $K > 1/2$, signaling the perfect conductance if $\Gamma=0$. For finite dissipation the conductance reaches a saturation value $\sim t^2 (\Gamma/\hbar \omega_c)^{1/K-2}$. Finally in region (d) we get

$$G \sim t^2 A_4(T) e^{-\sqrt{8\pi\Gamma/K^2 T}}, \quad \frac{(KE_c)^2}{\Gamma} \ll T \ll \Gamma \quad (44)$$

where $A_4(T)$ is again some power-law temperature-dependent function.

V. CONCLUSIONS

In the present paper we have calculated the conductance G of a dissipative Luttinger liquid with a quantum dot formed by two strong impurities, using an instanton approach. The following results have been obtained.

(i) Resonance tunneling survives in the presence of dissipation. However, unlike in the dissipation free case, the transport is strongly suppressed by the dissipation, and perfect transmission is not possible on the resonance. Instead, the conductance vanishes exponentially for small temperature and voltage [see Eqs. (38) and (42) with $\Delta=0$]. In comparison with the case of single impurity in the presence of dissipation, the transport on the resonance is still enhanced, which is manifested by a larger conductance with decreased exponential exponent by a factor 1/2.

(ii) Depending on the ratio of the Coulomb energy of the quantum dot, $|\Delta|E_c$, and the temperature T (or the voltage drop eV_0 , respectively), there is a crossover from cotunneling for low temperatures (or small applied voltage) to sequential tunneling for larger temperatures (or voltage). At resonance, $|\Delta|=0$, the region for cotunneling disappears completely and the conductance is always due to sequential tunneling. The

cross-over lines between cotunneling and sequential tunneling are given by Eqs. (29) and (32), respectively (compare also Fig. 1 and Fig. 2).

(iii) If the voltage drop through the impurities eV_0 is much smaller than all other energy scales, the response of the system is linear. Then for very weak dissipation, $\Gamma \ll KE_c$, and $1/2 < K < 1$, the conductance at resonance, $|\Delta|=0$, shows a minimum at $T \approx KE_c$ between the regimes (a) and (b) described by formulas (41) and (43). This result agrees with the findings of Furusaki and Nagaosa.⁶ For very low temperatures, $T < \Gamma$, however, the conductance drops exponentially due to the dissipation [see Eq. (42) and Fig. 3].

Off resonance, for $|\Delta|E_c > T$, the conductance is exponentially suppressed even at larger temperatures, see Eq. (43). This reduction in G is limited however by the crossover to cotunneling, see Eq. (33).

In the opposite limit of strong dissipation, $\Gamma \gg KE_c$, the conductance drops to exponentially small values as soon as $T \ll \Gamma$ [see Eqs. (42) and (44)].

(iv) At $T=0$ and at resonance, $\Delta=0$, the voltage dependent conductance $G=I/V_0$ shows a behavior similar to that of the temperature-dependent conductance, as follows from Eqs. (37)–(40). There is again a nonmonotonic behavior for $\Gamma < KE_c$ and a monotonic behavior for $\Gamma > KE_c$ (see also Fig. 3).

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APPENDIX A: LIST OF SYMBOLS

Symbol	Definition	Quantity
K		Luttinger liquid parameter
v		Excitation velocity
η		Dissipation strength
ω_c		High frequency cutoff
a		Spacing between impurities
V_0		External voltage
T		Temperature
t		Tunneling transparency
κ	$K/(\pi\hbar v)$	Compressibility
E_c	$\pi\hbar v/(Ka)$	Charging energy of the dot
Δ	$n + \frac{1}{2} - \frac{k_{ra}}{\pi}$	Distance to the resonance
Γ	$\hbar v K \eta$	Damping of plasmons
X	$E_c/(eV_0)$	
Y	$\Gamma/(KeV_0)$	
Z	$\hbar\omega_c/(KeV_0)$	
X_T	KE_c/T	
Y_T	Γ/T	
Z_T	$\hbar\omega_c/T$	

APPENDIX B: KINK-ANTIKINK INTERACTION

In this appendix we briefly describe how to calculate the instanton action for the interaction between the kink and antikink [i.e., $F(y)$ given by formula (24)]. The integrand of the integral on the right hand side of formula (24) shows several crossovers in Ω space. These crossovers can be obtained by equating Yy/Ω and the exponent of the exponential to 1, separately. For $Y \ll X$, the crossovers are at $\Omega = Yy$ and $\Omega = Xy$, which separates the three regions $\Omega \ll Yy \ll Xy$, $Yy \ll \Omega \ll Xy$, and $Yy \ll Xy \ll \Omega$. For $Y \gg X$, the crossovers are at $\Omega = Yy$ and $\Omega = X^2y/Y$, which again separates three regions $\Omega \ll X^2y/Y \ll Yy$, $X^2y/Y \ll \Omega \ll Yy$, and $X^2y/Y \ll Yy \ll \Omega$. To evaluate the integral we disassemble it into small ones according to the different regions separated by the crossovers. We consider the two cases $Y \ll X$ and $Y \gg X$, separately. We start with $Y \ll X$. For $1 \gg Yy \gg Yy$, we have

$$F(y) \approx \int_0^{Zy} d\Omega \frac{1 - \cos \Omega}{\Omega} = \ln(Zy). \quad (\text{B1})$$

For $Yy \ll 1 \ll Xy$, we have

$$F(y) \approx \frac{1}{2} \int_0^{Xy} d\Omega \frac{1 - \cos \Omega}{\Omega} + \int_{Xy}^{Zy} d\Omega \frac{1 - \cos \Omega}{\Omega} \\ = \frac{1}{2} \ln(Xy) + \ln\left(\frac{Z}{X}\right). \quad (\text{B2})$$

For $1 \ll yY \ll yX$, we have

$$F(y) \approx \frac{1}{2} \int_0^{Yy} d\Omega \frac{\sqrt{Yy\Omega}}{\Omega^2} (1 - \cos \Omega) + \frac{1}{2} \int_{Yy}^{Xy} d\Omega \frac{1 - \cos \Omega}{\Omega} \\ + \int_{Xy}^{Zy} d\Omega \frac{1 - \cos \Omega}{\Omega} \\ = \frac{1}{2} \left[\sqrt{2\pi}(\sqrt{Yy} - 1) + \ln\left(\frac{X}{Y}\right) + 2 \ln\left(\frac{Z}{X}\right) \right]. \quad (\text{B3})$$

Now we discuss the case $Y \gg X$. For $1 \gg Yy \gg X^2y/Y$, one also gets the result formula (B1). For $X^2y/Y \ll 1 \ll Yy$, one gets

$$F(y) \approx \int_0^{Yy} d\Omega \frac{\sqrt{Yy\Omega}}{\Omega^2} (1 - \cos \Omega) + \int_{Yy}^{Zy} d\Omega \frac{1 - \cos \Omega}{\Omega} \\ = \sqrt{2\pi}(\sqrt{Yy} - 1) + \ln\left(\frac{Z}{Y}\right). \quad (\text{B4})$$

For $1 \ll X^2y/Y \ll Yy$, one finds

$$F(y) \approx \frac{1}{2} \int_0^{X^2y/Y} d\Omega \frac{\sqrt{Yy\Omega}}{\Omega^2} (1 - \cos \Omega) + \int_{X^2y/Y}^{Yy} d\Omega \frac{\sqrt{Yy\Omega}}{\Omega^2} \\ \times (1 - \cos \Omega) + \int_{Yy}^{Zy} d\Omega \frac{1 - \cos \Omega}{\Omega} \\ = \frac{1}{2} \left[\sqrt{2\pi Yy} + \sqrt{2\pi} \left(\frac{Y}{X} - 2 \right) + 2 \ln\left(\frac{Z}{Y}\right) \right]. \quad (\text{B5})$$

Finally, the results can be summarized by

$$F(y) = \begin{cases} \ln(Zy), & \text{(i) } yY, yX \ll 1 \\ \frac{1}{2} \ln(Z^2 y/X), & \text{(ii) } yY \ll 1 \ll yX \\ \frac{1}{2} \left[\sqrt{2\pi Y y} + \ln\left(\frac{Z^2}{XY}\right) \right], & \text{(iii) } 1 \ll yY \ll yX \\ \sqrt{2\pi Y y} + \ln\left(\frac{Z}{Y}\right), & \text{(iv) } yX^2/Y \ll 1 \ll yY \\ \frac{1}{2} \left[\sqrt{2\pi Y y} + \sqrt{2\pi} \frac{Y}{X} + 2 \ln\left(\frac{Z}{Y}\right) \right], & \text{(v) } 1 \ll yX^2/Y \ll yY. \end{cases} \quad (\text{B6})$$

APPENDIX C: CROSSOVER BETWEEN SEQUENTIAL AND COTUNNELING

The saddle points for sequential and cotunneling are given by Eqs. (C1) and (C2), respectively.

$$y_s = \begin{cases} \frac{4}{1-2\Delta X}, & \text{(a) } Y+2\Delta X, X(1+2\Delta) \ll 1 \\ \frac{2}{1-2\Delta X}, & \text{(b) } Y+2\Delta X \ll 1 \ll X(1+2\Delta) \\ \frac{2\pi Y}{(1-2\Delta X)^2}, & \text{(c) } 1 \ll Y+2\Delta X \ll X(1+2\Delta) \\ \frac{8\pi Y}{(1-2\Delta X)^2}, & \text{(d) } X(1+2\Delta) \ll 1 \ll Y+2\Delta X \\ \frac{2\pi Y}{(1-2\Delta X)^2}, & \text{(e) } 1 \ll X(1+2\Delta) \ll Y+2\Delta X. \end{cases} \quad (\text{C1})$$

Here the different areas of validity (a)–(e) are separated by the lines $X=Y$, $Y=1-2\Delta X$ and $X=1/(1+2\Delta)$ (see also Fig. 1). These saddle points have to be compared with the saddle points for cotunneling

$$y_c = \begin{cases} 4, & \text{(a')} Y, X \ll 1 \\ 2, & \text{(b')} Y \ll 1 \ll X \\ 2\pi Y, & \text{(c')} 1 \ll Y \ll X \\ 8\pi Y, & \text{(d')} X \ll 1 \ll Y \\ 2\pi Y, & \text{(e')} 1 \ll X \ll Y, \end{cases} \quad (\text{C2})$$

with the areas of validity separated by the lines $X=Y$, $Y=1$, and $X=1$ (see again Fig. 1). Clearly, for $\Delta=0$ both sets of saddle points are identical, but the action of the cotunneling process is always larger than that of sequential tunneling and hence sequential tunneling prevails. Qualitatively, this remains true for small but finite $\Delta X \ll 1$. However, if ΔX

becomes of the order one, the saddle points for sequential tunneling move to larger values such that for $X > X_c(Y)$ cotunneling sets in.

The crossover between the sequential tunneling and the cotunneling is defined as the point at which the currents for the two different tunneling mechanisms are equal. Assuming $\Delta \ll 1$, in Sec. II E it is pointed out that the crossover between the cotunneling and the sequential tunneling in regimes (a) and (d) [i.e., $X \gg 1/(1+2\Delta)$] cannot happen. This is clearly illustrated in Fig. 1.

We start with very small Y (i.e., $Y < Y_1$) such that near the crossover the currents for the sequential tunneling and the cotunneling are given by formula (39) for regime (b) and Eq. (35) for regime (b'), respectively. The value of Y_1 will be determined afterward. After a straightforward calculation the crossover is found to be

$$X = \frac{1 - \left(\frac{t^{2K}}{2\Delta Z^2}\right)^{1/(1-K)}}{2\Delta} \approx \frac{1}{2\Delta}. \quad (\text{C3})$$

However, this result is self-consistent only if the current for the sequential tunneling is indeed given by formula (39) for regime (b) [i.e., $Y+2\Delta X \ll 1 \ll X(1+2\Delta)$]. This leads to a restriction on the validity of Eq. (C3)

$$Y \ll Y_1 \equiv \left(\frac{t^{2K}}{2\Delta Z^2}\right)^{1/(1-K)}. \quad (\text{C4})$$

Y_1 is essentially very small.

For Y slightly bigger than Y_1 , the current for the sequential tunneling is given by formula (38) for regime (c), while the current for the cotunneling remains in regime (b'). In this situation the crossover is determined by

$$\begin{aligned}
& t^2 \omega_\delta \left(\frac{Y}{Z}\right)^{(2/K)+(1/2)} \left(\frac{X}{Y}\right)^{1/K} Z^{3/2} (1-2\Delta X)^{-3/2} \\
& \quad \times \exp\left(-\frac{\pi Y}{K(1-2\Delta X)}\right) \\
& = t^4 \omega_\delta \left(\frac{X}{Z}\right)^{2/K} \left(\frac{1}{Z}\right)^{(2/K)-1}, \tag{C5}
\end{aligned}$$

where we have chosen to express the currents in terms of the ratios of relevant energy scales X , Y , and Z . After some algebra we obtain

$$X \approx \frac{1 - \frac{\hbar \pi Y}{2KS_{\text{kink}}}}{2\Delta} \approx \frac{1}{2\Delta}. \tag{C6}$$

This result is valid for $Y_1 \ll Y \ll 1$.

Now for $1 \ll Y \ll X$, the current for the sequential tunneling is still in regime (c), while the one for the cotunneling just moves into regime (c') and is given by formula (34). In this regime the crossover is given by

$$\begin{aligned}
& t^2 \omega_\delta \left(\frac{Y}{Z}\right)^{2-K+1/2} \left(\frac{X}{Y}\right)^{1/K} Z^{3/2} (1-2\Delta X)^{-3/2} \\
& \quad \times \exp\left(-\frac{\pi Y}{K(1-2\Delta X)}\right) \\
& = t^4 \omega_\delta \left(\frac{Y}{Z}\right)^{(4/K)+(1/2)} \left(\frac{X}{Y}\right)^{2/K} Z^{3/2} \exp\left(-\frac{2\pi Y}{K}\right), \tag{C7}
\end{aligned}$$

which leads to

$$1 - 2\Delta X = \frac{\pi Y}{\frac{2KS_{\text{kink}}}{\hbar} + 2\pi Y + \ln\left(\frac{Z^3}{XY^2}\right)} \approx \frac{\pi Y}{\frac{2KS_{\text{kink}}}{\hbar} + 2\pi Y}. \tag{C8}$$

This crossover implies that for $1 \ll Y \ll X$ the range of X on the crossover is within $1/(4\Delta) \ll X \ll 1/(2\Delta)$. Thus, for $KS_{\text{kink}}/\hbar \ll 1/\Delta$ the crossover intersects with the line $Y=X$ at

$Y=X \approx 1/(4\Delta)$; otherwise they meet at $Y=X \approx 1/(2\Delta)$.

For $Y \gg X$, the current for the sequential tunneling and the cotunneling are given by formula (38) for regime (e) and Eq. (34) for regime (e'). The crossover is given by

$$\begin{aligned}
& t^2 \omega_\delta \left(\frac{Y}{Z}\right)^{(2/K)+(1/2)} Z^{3/2} (1-2\Delta X)^{-3/2} \exp\left(-\frac{\pi Y}{K(1-2\Delta X)}\right) \\
& \quad \times \exp\left(-\frac{\sqrt{2\pi Y}}{KX}\right) \\
& = t^4 \omega_\delta \left(\frac{Y}{Z}\right)^{(4/K)+(1/2)} Z^{3/2} \exp\left(-\frac{2\pi Y}{K}\right) \exp\left(-\frac{2\sqrt{2\pi Y}}{KX}\right). \tag{C9}
\end{aligned}$$

This leads to

$$\begin{aligned}
1 - 2\Delta X &= \frac{\pi Y}{\frac{2KS_{\text{kink}}}{\hbar} + 2\pi Y + \sqrt{2\pi \frac{Y}{X}} + 2 \ln\left(\frac{Z}{Y}\right)} \\
&\approx \frac{\pi Y}{\frac{2KS_{\text{kink}}}{\hbar} + 2\pi Y}. \tag{C10}
\end{aligned}$$

Finally, the crossover between the sequential tunneling and the cotunneling can be summarized as formula (29). The various regimes and the crossovers between them are illustrated in XY plane in Fig. 1.

APPENDIX D: CURRENTS AT FINITE TEMPERATURES

In this appendix we quote the final result for the function $F_T(z)$, which first appears in Eq. (27). It reads

$$F_T(z) = \begin{cases} \ln(Z_T z), & \text{(i) } zY_T, zX_T \ll 1 \\ \frac{1}{2} \ln(Z_T^2 z / X_T), & \text{(ii) } zY_T \ll 1 \ll zX_T \\ \frac{1}{2} \left[\sqrt{2\pi Y_T z} + \ln\left(\frac{Z_T^2}{X_T Y_T}\right) \right], & \text{(iii) } 1 \ll zY_T \ll zX_T \\ \sqrt{2\pi Y_T z} + \ln\left(\frac{Z_T}{Y_T}\right), & \text{(iv) } zX_T^2 / Y_T \ll 1 \ll zY_T \\ \frac{1}{2} \left[\sqrt{2\pi Y_T z} + \sqrt{2\pi \frac{Y_T}{X_T}} + 2 \ln\left(\frac{Z_T}{Y_T}\right) \right], & \text{(v) } 1 \ll zX_T^2 / Y_T \ll zY_T. \end{cases} \tag{D1}$$

In the case of finite temperature, the previous saddle-point solution for zero temperature and finite voltage does not apply when the distance between the kink and antikink is larger than the size of the imaginary time axis. Thus the maximum instanton action occurs at $\tau = \hbar/T$, i.e., $z=1$. Then the tunneling rate can be approximated as proportional to

$$\exp[-S(z_1 = 1, z_2 = 0)/\hbar]$$

for the sequential tunneling and

$$\exp[-S(z_1 = 1, z_2 = 1)/\hbar]$$

for the cotunneling, respectively. Unlike in the case of zero temperature, the tunneling rates along both the voltage-favored and -unfavored directions are comparable. Therefore, the current should be proportional to the difference between these two. To the lowest order of eV_0 , we get

$$I \sim eV_0 t^2 \exp\left[-\frac{2}{K}F_T(1) - \frac{\Delta X_T}{K}\right] \quad (\text{D2})$$

for the sequential tunneling and

$$I \sim eV_0 t^4 \exp\left[-\frac{4}{K}F_T(1)\right] \quad (\text{D3})$$

for the cotunneling. This approximation is not accurate enough to give correct power-law temperature dependence of the current. However, in the dissipative regime it captures the dominant exponential temperature dependence. Plugging the expressions of $F_T(1)$ into the above two formulas, we obtain the results given in Secs. III B and IV B.

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